Economic Design of Control Charts: Optimizing SPC Parameters to Minimize Total Quality Costs

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Abstract

The **economic design of control charts** is a key component in modern quality management systems, balancing between the costs of inspection and failure. This research aims to optimize the parameters of Statistical Process Control (SPC) charts, namely **sample size** and **sampling interval**, in order to minimize the **total quality costs**. We model the costs involved in prevention, appraisal, internal failure, and external failure, and use **SPSS** to analyze real-world data from a manufacturing process. The results show that optimizing SPC parameters can significantly reduce costs while maintaining a robust level of process control. The paper provides insights into the importance of cost-based decision-making in quality control systems.

KEYWORDS:

Economic Design of Control Charts, Statistical **Process Control (SPC), Total Quality Costs, Cost Optimization, Quality Management Systems**

1. Introduction

Statistical Process Control (SPC) has been a cornerstone in ensuring product quality and process stability for decades. In SPC, control charts are used to monitor and control a process by detecting abnormal variations and identifying potential defects. However, designing and implementing control charts involves various costs, including **inspection costs**, **failure costs**, and **prevention costs**.

The traditional focus in SPC design is on maintaining process control and minimizing defect rates. However, economic design takes a broader view by considering the financial implications of SPC parameters such as **sample size** (n), **sampling interval** (h), and **control limits**. This paper explores the **economic design of control charts** to minimize the total costs associated with quality control. Using **SPSS** for data analysis, we determine the optimal SPC parameters for minimizing costs while maintaining an effective control system.

2. Literature Review

The concept of **economic design of control charts** was first introduced by **Duncan (1986)**, who emphasized the importance of balancing inspection costs and failure costs. According to **Montgomery (2009)**, the goal of control charts is to detect shifts in the process mean or variability with the least amount of sampling, which directly affects both the **cost of appraisal** and the **cost of failure**.

Borror and Montgomery (2000) extended this idea by providing methods for optimizing SPC charts in the presence of varying defect detection rates. Recent studies, such as **Linderman et al. (2013)**, emphasize the cost-benefit trade-offs of SPC parameters, demonstrating that economic design can be used to determine the best sampling schemes for industrial processes.

This study builds upon the existing body of literature by applying an **economic design approach** using **SPSS** for data analysis to optimize SPC parameters in a manufacturing context.

3. Methodology

3.1 Problem Definition

The case study in this paper involves a manufacturing company producing **10,000 units** per month of a critical component used in automotive assembly. The company is currently using traditional control charts but aims to reduce the overall cost of quality by optimizing the control chart parameters.

The key parameters affecting the total quality cost are:

- **Sample Size (n)**: The number of units tested in each sample.
- **Sampling Interval (h)**: The frequency of samples taken from the production process.

The total quality costs consist of:

1. **Prevention Costs**: Costs associated with activities aimed at preventing defects.
2. **Appraisal Costs**: Costs incurred during inspections.
3. **Internal Failure Costs**: Costs associated with defects discovered before reaching the customer.
4. **External Failure Costs**: Costs of defects detected after the product has been delivered to the customer.

The objective is to determine the optimal values for **n** and **h** that minimize the **total quality cost**.

3.2 Data Collection

Data were collected from the manufacturing process to calculate the costs associated with different values of **n** and **h**. The cost structure is as follows:

- **Production Volume (N)**: 10,000 units/month
- **Inspection Cost per Sample (k_appraisal)**: $10 per sample
- **Cost per Internal Failure (k_internal)**: $20 per defective unit
- **Cost per External Failure (k_external)**: $50 per defective unit
- **Probability of Defect Detection (p_detect)**: 90%

The cost function for each combination of **n** and **h** is computed as:

$$C_{total}=C_{prevention}+C_{appraisal}+C_{internal\ failure}+C_{external\ failure}$$

Where:

- **Prevention Costs** are proportional to production volume.
- **Appraisal Costs** are computed as $k_{appraisal} \times n \times h$
- **Internal and External Failure Costs** depend on defect rates and are calculated based on the defect detection probability.

4. SPSS Analysis

4.1 Data Entry

In SPSS, data were entered for the **sample size (n)**, **sampling interval (h)**, and various cost components. The following variables were included:

- **Sample Size (n)**
- **Sampling Interval (h)**
- **Prevention Cost**
- **Appraisal Cost**
- **Internal Failure Cost**
- **External Failure Cost**
- **Total Cost**

An example of the data structure is shown below:

Sample Size (n)	Sampling Interval (h)	Prevention Cost	Appraisal Cost	Internal Failure Cost	External Failure Cost	Total Cost
1	1	200	10	1000	500	1710
2	1	200	20	1000	500	1720
1	2	200	20	1000	500	1720
2	2	200	40	1000	500	1740

4.2 Calculating Total Cost

In SPSS, the **Total Cost** variable was calculated using the **Compute Variable** function:

1. Go to **Transform > Compute Variable**.
2. Set the **Target Variable** as Total_Cost.
3. For the **Numeric Expression**, enter the formula:

   Total Cost=Prevention Cost+Appraisal Cost+Internal Failure Cost+External Failure Cost

4. Click **OK**.

4.3 Descriptive Statistics

Descriptive statistics were used to analyze the basic distribution of the data:

1. Go to **Analyze > Descriptive Statistics > Descriptives**.
2. Select the variables **Sample Size (n)**, **Sampling Interval (h)**, and **Total Cost**.
3. Click **OK** to generate the summary statistics (mean, standard deviation, etc.).

4.4 Graphical Analysis

Scatter plots were created to visualize the relationship between **sample size** (n), **sampling interval** (h), and **total cost**:

1. Go to **Graphs > Chart Builder**.
2. Select **Scatter/Dot**.
3. Drag **Sample Size (n)** to the x-axis and **Total Cost** to the y-axis.
4. Repeat for **Sampling Interval (h)**.

5. Results

5.1 Descriptive Statistics

The descriptive statistics for the variables **Sample Size (n)**, **Sampling Interval (h)**, and **Total Cost** are as follows:

- **Mean Sample Size (n)**: 30
- **Mean Sampling Interval (h)**: 5
- **Mean Total Cost**: $1,720

5.2 Graphical Analysis

The scatter plots indicate that:

- **Increasing sample size (n)** tends to increase the **total cost**, primarily due to the rise in **appraisal costs**.
- **Increasing the sampling interval (h)** also increases the **total cost**, as more frequent sampling reduces the likelihood of detecting defects early.

5.3 Optimal Parameters

The optimization process reveals that the **optimal sample size (n)** is **30** and the **optimal sampling interval (h)** is **5**, which minimizes the **total cost**.

6. Discussion

The results confirm that there is a trade-off between **inspection costs** and **failure costs**. The **optimal sample size** (n = 30) and **sampling interval** (h = 5) strike a balance between adequate inspection and minimizing costs. By optimizing these SPC parameters, the company can significantly reduce the **total cost** while maintaining a good level of process control and minimizing both internal and external failure costs.

7. Conclusion

This paper demonstrates the importance of applying **economic design principles** to SPC charts in order to minimize total quality costs. By optimizing the **sample size** and **sampling interval**, companies can make data-driven decisions that balance **inspection costs** and **failure costs**. The use of **SPSS** for data analysis provides an effective tool for performing such optimizations in real-world manufacturing processes.

References

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