Theoretical Introduction of Orbital Mechanics and Geometry of Satellite Orbits

Dimov Stojce Ilcev
University of Johannesburg (UJ), Johannesburg, South Africa

Abstract: In this paper introduces the theory of orbital mechanics and the geometry of satellite orbits as the basic fundamental laws of the planets and artificial platforms motion during their rotation around the Sun and Earth. Basic geometric theory of satellite coordinates is applied to determine the geographical azimuth and elevation (spacecraft altitude) angles required to point mobile satellite tracking antenna to the Geostationary Earth Orbit (GEO) or Non-GEO communication satellites. The introduced mathematical treatment takes into consideration circular and elliptical orbits, describes Satellite Look Angles, Track and Geometry in the Space and their significance with regard to the spacecraft use for Mobile Satellite Communications (MSC) systems. During more than four decades, commercial MSC networks have utilized GEO satellites extensively to the point where orbital portions have become crowded; coordination between satellites is becoming constrained and could never solve the problem of polar coverage. However, other Non-GEO MSC solutions have recently grown in importance because of their orbit characteristics and coverage capabilities in high latitudes and Polar Regions. The important parameters of the Satellite Look Angles such as Elevation and Azimuth angles and Satellite Track and Geometry such as Longitude and Latitude values are described.

Key Words: GEO, Non-GEO, MSC, Longitude, Latitude, MES, GNSS, TT&C, Perigee, Apogee, NOAA, LES, MES, Elevation, Azimuth, Sub Satellite Point, SES, AES, Doppler Effect

1. Introduction

The satellite platform is located in orbit around the Earth at different altitudes, starting from 20 km in the stratosphere and up to 36,000 km in the Space. Orbital mechanics is a specific discipline describing planetary and satellite motions in the Solar system, which can solve the problems of calculating the look angles by the elevation and azimuth amounts, position by the longitude and latitude values, speed, path, perturbation and other orbital values of planets and satellites.

Thus, these parameters are very important for the tracking satellite in focus of directional satellite antenna onboard Mobile Earth Station (MES). The space segment of an artificial satellite system is one of its three major operational components, the others being the ground and user segments. It comprises the different satellite constellations, and the satellite uplink and downlink. Namely, here are introduced the fundamental laws governing satellite orbits and the principal parameters that describe the motion of the Earth's artificial satellites. The types of satellite orbits are also classified and compared from the MSC system viewpoint in terms of coverage and link performances.

The satellite Bus and payloads for communications, broadcasting and navigation (GNSS) are discussed. The chapter concludes with a brief overview of satellite launch vehicles and orbit insertion. Types of satellite orbits and perturbations are also classified and compared from the communication and navigation systems viewpoint in terms of coverage and link performances.

During the last four decades military and commercial MSC systems and networks have utilized GEO extensively to the point where orbital portions have become quite crowded, coordination between satellites is becoming constrained and could never solve the problem of polar coverage. On the other hand, Non-GEO MSC solutions have recently grown in importance because of their orbit characteristics and coverage capabilities in high latitudes and Polar Regions.

The satellite service begins when a spacecraft is located as a space platform in the desired orbital position in a space environment around the Earth. This space environment is a very specific part of the Universe, where many factors and determined elements affect the planet and satellite motions. The Earth is surrounded by a thick layer of many different gasses in atmosphere and parts known as the atmosphere, whose density decreases as the altitude increases. Hence, there is no air and the atmosphere disappears at about 180 km above the Earth, where the Cosmos begins. The endless environment in space is not very friendly and is extremely destructive, mainly because there is no atmosphere, the cosmic radiation is very powerful, the vacuum creates very high pressure on spacecraft or other bodies and there is the negative influence of very low temperatures.

The Earth's gravity keeps everything on its surface. All the heavenly bodies such as the Sun, Moon, planets and stars have gravity and reciprocal reactions. Any object flying in the atmosphere continues to travel until it meets forces due to the Earth's gravity or until it has enough speed to surpass gravity and to hover in the stratosphere. However, to send an object into space, it first has to overcome gravity and then travel at least at a particular minimum speed to stay in space. In this case, an object traveling at about 5 miles/sec can circle around the Earth and become an artificial spacecraft.
An enormous amount of energy is necessary to put a satellite into orbit and this is realized by using a powerful rockets or launchers, which are defined as an apparatus consisting of a case containing a propellant (fuel) and reagents by the combustion of which it is projected into the space. As the payload is carried on the top, the rocket is usually separated and drops each stage after burnout and brings a payload up to the required velocity and leaves it in orbit. A rocket is also known as a booster, as a rocket starts with a low velocity and attains some required height, where air drag decreases and it attains a higher velocity.

2. Platforms and Orbital Mechanics

The platform is an artificial object located in orbit around the Earth at a minimum altitude of about 20 km in the stratosphere and a maximum distance of about 36,000 km in the Space. The artificial platforms can have a different shape and designation but usually they have the form of aircraft, airship or spacecraft. In addition, there are special space stations and space ships, which are serving on more distant locations from the Earth's surface for scientific exploration and research and for cosmic expeditions.
Orbital mechanics is a specific discipline describing planetary and satellite motion in the Solar system, which can solve the problems of calculating and determining the position, speed, path, perturbation and other orbital parameters of planets and satellites. In fact, a space platform is defined as an unattended object revolving about a larger one. Although it was used to denote a planet's Moon, since 1957 it also means a man-made object put into orbit around a large body (planet), when the former USSR launched its first spacecraft Sputnik-1. Accordingly, man-made satellites are sometimes called artificial satellites.
Orbital mechanics support a communication satellites project in the phases of orbital design and operations. The orbital design is based on a generic survey of orbits and at an early stage in the MSC project is tasked to identify the most suitable orbit for the objective MSC service. The orbital operation is based on rather short-term knowledge of the orbital motion of the satellite and starts with TT&C maintenances after the satellite is located in orbit. In effect, only a few types of satellite orbits are well suited for MSC and navigation systems.

3. Laws of Satellite Motion

A satellite is an artificial object located by rocket in space orbit following the same laws in its motion as the planets rotating around the Sun. Thus, Johannes Kepler, a German mathematician, has contributed a great deal to the field of astronomy and astrology. The Laws of Planetary Motion formulated by Kepler proves that the orbits of the planets are ellipses and not circles, as believed by many. The ellipse is a geometrical shape that has two foci, such that, the sum of the distance from the focus to any point on the surface of the ellipse is constant. The orbits of planets have small eccentricities (flattening of ellipse), and so, they appear as circles. Based on the properties of ellipses, Johannes Kepler devised three laws that explain the motion of planets around the Sun.
A satellite is an artificial object launched and located by rocket in orbit follows the same laws in its motion as the planets rotating around the Sun. Thus, three important laws for planetary motion were derived by Johannes Kepler, as follows:
1. First Law – The first law is also known as The Law of Orbits. As stated, an ellipse has two foci. While studying the motion of planets around the Sun, Kepler explained that the path followed was elliptical, with the Sun as one of the two foci. In simple terms, the law is stated as:

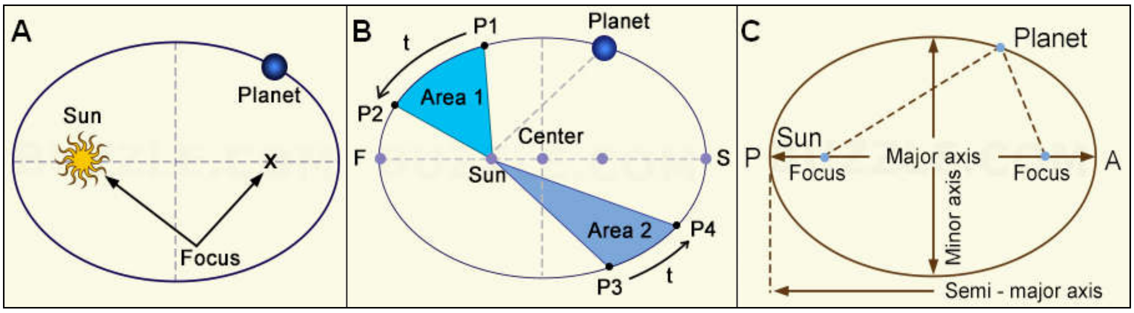

Figure 1. Kepler's Laws of Satellite Motion

The orbit of each planet follows an elliptical path or all planets move in elliptical orbits, with the Sun at one focus, which is shown in **Figure 1 (A).** This indicates that the Sun is one focus, while the other focus is known as the vacant or empty focus. As seen in the diagram, the Sun and the empty focus lie on the major axis of the ellipse, and the planet lies on the surface of the ellipse. As the planet is continuously moving around the Sun, and as the Sun is not at the center of the ellipse, the Planet-Sun distance will always keep on changing. The Law of Orbits proofs that planet motion lies in the plane around the Sun (1602).

2. Second Law – The second law is also known as The Law of Equal Areas, shown in **Figure 1 (B).** As the Sun is one of the foci, it is clear that the Planet-Sun distance will be changing. But, the planet covers up for the increase in the distance by moving faster when it is closer to the Sun. This indicates that planets do not move at an uniform speed. This law states that: The line from the Sun to orbital planet or radius vector (r) sweeps out equal areas in equal intervals of time (t) as the planet travels around the ellipse. The point at which the speed of the planet is fastest is known as Perihelion or Perigee indicated with F (Fastest motion), while the distance with slowest speed is known as Aphelion (Apogee) indicated by S (Slowest motion). The distance measured from the Perihelion to the position of the Sun is known as Perihelion distance, while the distance from the Sun to the Aphelion is known as the Aphelion distance. The law says that, while moving in an elliptical path, the planet moves faster when it is closer to the Sun. This way, the radius sweeps equal areas in equal amount of time. If the planet is observed at successive times (P1, P2, P3, P4), it draw the radius vector during the first second observations, showing that the two radius vectors having the same area. So, the area swept during the time (t) by the planet to move from P1 to P2 is the same as the area swept while moving from P3 to P4. This is the Law of Equal Areas (1605).

3. Third Law – The third law of planetary motion in ellipse with Perigee (P) and Apogee (A) is alternatively known as The Law of Periods and Harmonic Law, see **Figure 1 (C).** This law relates the time required by a planet to make a complete trip around the Sun to its mean distance from the Sun. It can be simply stated as: The square of the planet orbital period is directly proportional to the cube of the semi-major axis of its orbit. The square of the planet's orbital period around the Sun (T) is proportional to the cube of the semi-major axis (a = distance from the Sun) of the ellipse for all planets in the Solar system (1618).

Kepler's laws only describe the planetary motion if the mass of central body insofar as it is considered to be concentrated in its centre and when its orbits are not affected by other systems. However, these conditions are not completely fulfilled in the case of Earth motion and its artificial satellites. Namely, the Earth does not have an ideal spherical shape and the different layers of mass are not equally concentrated inside of the Earth's body. Because of this, the satellite motions are not ideally synchronized and stable, so the motions are namely slower or faster at particular orbital sectors, which present certain exceptions to the rule of Kepler's Laws.

Furthermore, in distinction from natural satellites, whose orbits are almost elliptical, the artificial satellites can also have circular orbits, for which the basic relation can be obtained by the equalizing the centrifugal and centripetal Earth forces.

Kepler's laws only describe the planetary motion if the mass of central body insofar as it is considered to be concentrated in its centre and when its orbits are not affected by other systems. These conditions are not completely fulfilled in the case of Earth motion and its artificial satellites.

The Earth does not have an ideal spherical shape and the different layers of mass are not equally concentrated inside of the Earth's body. Because of this, the satellite motions are not ideally synchronized and stable, thus the motions are namely slower or faster at particular orbital sectors, which presents certain exceptions to the rule of Kepler's Laws.
Kepler's Laws were based on observational records and only described the planetary motion without attempting an additional theoretical or mathematical explanation of why the motion takes place in that manner.
In 1687, the English physicist British Sir Isaac Newton published his breakthrough work "Principia Mathematica" with own syntheses, known as the Three Laws of Motion, such as follows:
1. Law I – Every body continues in its state of rest or uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it.
2. Law II – The change of momentum per unit time of a body is proportional to the force impressed on it and is in the same direction as that force.
3. Law III – To every action there is always an equal and opposite reaction.
On the basis of Law II, Newton also formulated the Law of Universal Gravitation, which states that any two bodies attract one another with a force proportional to the products of their masses and inversely proportional to the square of the distance between them. This law may be expressed mathematically for a circular orbit with the relations:

$$F = m\,(2\pi/t)^2\,(R + h) = G\,[M\cdot m/(R + h)^2] \quad (1)$$

where parameter m = mass of the satellite body; t = time of satellite orbit; R = equatorial radius of the Earth (6.37816×10^6 m); h = altitude of satellite above the Earth's surface; G = Universal gravitational constant (6.67×10^{-11} N m^2/kg^{-2}); M = Mass of the Earth body (5.976032×10^{24} kg) and finally, F = force of mass (m) due to mass (M). Force of mass can be also presented by the following relation:

$$F = ma = dv/dt \quad (2)$$

here a = acceleration and v = velocity of satellite orbit. The force of attraction between two distant point masses m_1 and m_2 separated by a distance *r* is giving the following relation:

$$F = Gm_1m_2/r^2 \quad (3)$$

Where G = Newtonian (or universal) gravitation constant.
Consider the simple circular orbit and assuming that the Earth is a sphere, it is possible that can be treated a point mass. The centripetal force F_c required to keep the satellite in a circular orbit = mv^2/r, where v = orbital velocity of the satellite.

The force of gravity that supplies this centripetal force is GMm/r^2, where M = mass of the Earth and m is the mass of the satellite. Equating the two forces gives relation:

$$F_c = mv^2/r = GMm/r^2 \quad (4)$$

Division by m eliminates the mass of the satellite from the equation, which means that the orbit of a satellite is independent of its mass. Thus, the period of the satellite is the orbit circumference divided by the velocity: $T = 2\pi r/v$. Substituting in equation 3.3 gives the following relation:

$$T^2 = (4\pi^2/GM)\,r^3 \quad (5)$$

The first generation of the NOAA meteorological satellites orbit at approximately 850 km above the Earth's surface. Since the equatorial radius of the Earth is about 6378 km, the orbit radius is about 7228 km. In fact, substituting in equation 4 shows that the NOAA satellites have a period of about 102 min.

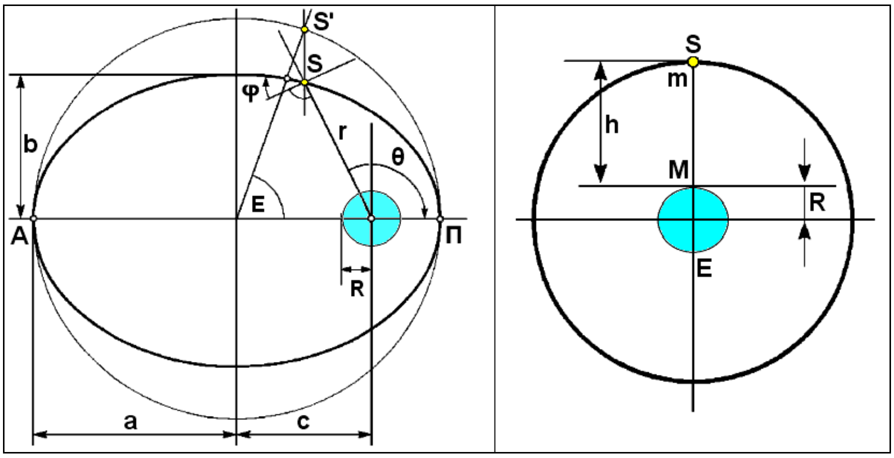

Figure 2. Elliptical and Circular Satellite Orbits

However, radius required for a satellite in GEO has the same angular velocity as the Earth, so the angular velocity mean motion constant of a satellite shows:

$$\xi = 2\pi/T \quad (6)$$

Substituting equation 6 in equation 5 is giving the following formula:

$$r^3 = GM/\xi^2 \quad (7)$$

Inserting the angular velocity of the Earth, the required radius for a GEO is 42,164 km or about 35,786 km above the Earth's surface.

3.1. Geometry of Elliptical Orbit

The satellite in circular orbit undergoes its revolution at a fixed altitude and with fixed velocity, while a satellite in an elliptical orbit can drastically vary its altitude and velocity during one revolution. The elliptical orbit is also subject to Kepler's Three Laws of satellite motion.
Therefore, the characteristics of elliptical orbit can be determined from elements of the ellipse of the satellite plane with the perigee (Π) and apogee (A) and its position in relation to the Earth, see **Figure 2 (Left).** The parameters of elliptical orbit are presented as follows:

$$e = c/a = \sqrt{[1 - (b/a)^2]} \quad \text{or} \quad e = (\sqrt{a^2} - b^2/a) \qquad p = a\,(1 - e^2) \quad \text{or} \quad p = b^2/a$$

$$c = \sqrt{(a^2 - b^2)} \qquad a = p/1 - e^2 \qquad b = a\,\sqrt{(1 - e^2)} \quad (8)$$

Where e = eccentricity, which determines the type of conical section; a = large semi-major axis of elliptical orbit; b = small semi-major axis of elliptical orbit; c = axis between centre of the Earth and centre of ellipse and p = focal parameter. However, the equation of ellipse derived from polar coordinates can be presented with the resulting trajectory equation as follows:

$$r = p/1 + e \cos \Theta \quad [\text{m}] \quad (9)$$

Where values r = distance of the satellites from the centre of the Earth (r = R+h) or radius of path; Θ = true anomaly and E = eccentric anomaly. In this case, the real position of the satellite will be determined by the angle called "the true anomaly", which can be counted positively in the direction of movement of the satellite from 0° to 360°, between the direction of the perigee and the direction of the satellite (S).

The position of the dedicated satellite can also be defined by eccentric anomaly, which is the argument of the image in the mapping, which transforms the elliptical trajectory into its principal circle, an angle counted positively in the direction of movement of the satellite from 0 to 360º, between the direction of the perigee and the direction of the satellite. The relations for both mentioned anomalies are given by the following equations:

$$\cos \Theta = \cos E - e/1 - e \cos E \qquad \cos E = \cos \Theta + e/1 + e \cos \Theta \tag{10}$$

The total obtained mechanical energy of a satellite in elliptical orbit is constant; although there is an interchange between the potential and the kinetic energies. As a result, a satellite slows down when it moves up and gains speed as it loses height. Thus, considering the termed gravitation parameter μ=GM (Kepler's Constant μ=3.99 x 105 km^3/sec^2), the velocity of a satellite in an elliptical orbit may be obtained from the following relation:

$$v = \sqrt{[GM (2/r) - (1/a)} = \sqrt{\mu (2/r) - (1/a)]} \tag{11}$$

Applying Kepler's Third Law the sidereal time of one revolution of the satellite in elliptical orbit is as follows:

$$t = 2\pi \sqrt{(a^3/GM)} = 2\pi \sqrt{(a^3/\mu)}$$

$$t = 3.147099647 \sqrt{(26{,}628.16 \bullet 103)^3 \bullet 10^{-7}} = 43{,}243.64 \quad [s] \tag{12}$$

Therefore, the last equation is the calculated period of sidereal day for the elliptical orbit of Russian satellite Molniya with value of apogee = 40,000 km, perigee = 500 km, revolution time = 719 min and a = 0.5 (40,000 + 500 + 2 x 6,378.16) = 26,628.15 km.

3.2. Geometry of Circular Orbit

The circular orbit is a special case of elliptical orbit, which is formed from the relations a = b = r and e = 0, see **Figure 2 (Right).** According to Kepler's Third Law, the solar time (τ) in relation with the right ascension of an ascending node angle (Ω); the sidereal time (t) with the consideration that μ=GM and satellite altitude (h), for a satellite in circular orbit will have the following relations:

$$\tau = t /(1 - \Omega t / 2\pi)$$

$$t = 2\pi \sqrt{(r^3/\mu)} = 3.147099647 \sqrt{(r^3 \cdot 10^{-7})} \quad [s]$$

$$h = [\sqrt[3]{(\mu t^2/4\pi^2)}] - R = 2.1613562 \cdot 10^4 (\sqrt[3]{t^2}) - 6.37816 \cdot 10^6 \ [m] \tag{13}$$

The time is measured with reference to the Sun by solar and sidereal day. Thus, a solar day is defined as the time between the successive passages of the Sun over a local meridian. In fact, a solar day is a little bit longer than a sidereal day, because the Earth revolves by more than 360º for successive passages of the Sun over a point 0.986º further.

On the other hand, a sidereal day is the time required for the Earth to rotate one circle of 360º around its axis: t_E = 23 h 56 min 4.09 sec. Therefore, a geostationary satellite must have an orbital period of one sidereal day in order to appear stationary to an observer on Earth.

During rotation the duration of sidereal day t = 85,164,091 (s) and is considered in such a way for synchronous orbit that h = 35,786.04x10^3 (m). The speed of satellite is conversely proportional to the radius of the path (R+h) and for the satellite in circular orbit it can be calculated from the following relation:

$$v = \sqrt{(MG/R + h)} = \sqrt{(\mu/r)} = 1.996502 \cdot 10^{-7}/\sqrt{r} = 631.65 \sqrt{r} \quad [m/s] \tag{14}$$

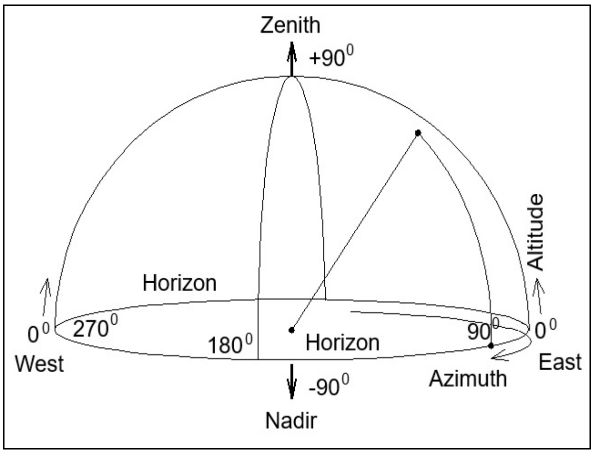

Figure 3. The Azimuth, Altitude and Zenith

From equation (2.8.) and using the duration of sidereal day (t_E) gives the relation for the radius of synchronous or geostationary orbits:

$$r = \sqrt[3]{[(\mu t) / 2\pi)^2]} \quad (15)$$

The satellite trajectory can have any angle of orbital planes in relation to the equatorial plane: in the range from PEO up to GEO plane. Namely, if the satellite is rotating in the same direction of Earth's motion, where (t_E) is the period of the Earth's orbit, the apparent orbiting time (t_a) is calculated by the following relation:

$$t_a = t_E \cdot t/t_E - t \quad (16)$$

This means, inasmuch as $t = t_E$ the satellite is geostationary ($t_a = \infty$ or τ=0). In **Table 1** several values for times different than synchronous orbital time are presented. According to **Table 1** and equation (9) it is evident that a satellite does not depend so much on its mass but decreases with higher altitude. In addition, satellites in circular orbits with altitudes of a 1,700, 10,400 and 36,000 km, will have t/ τ values 2/2,18, 6/8 and 24/zero, respectively. In this case, it is evident that only a satellite constellation at altitudes of about 36,000 km can be synchronous or geostationary.

Table 1. The Values of Times Different than the Synchronous Time of Orbit

Parameter	Values of time					Unit
t	86,164.00	43,082.05	21,541.23	10,770.61	6,052.00	s
h	35,786.00	20,183.62	10,354.71	4,162.89	800.00	km
(R+h)	42,164.00	26,561.78	16,732.87	10,541.05	7,178.00	km
v	3,075.00	3,873.83	4,880.72	5,584.12	7,450.00	km/s^{-1}

4. Horizon and Geographic Satellite Coordinates

The horizon system is a type of orbital coordinate parameters that can be used to locate the position of objects in the space. In a satellite orbits are usually using local geographic coordinates, which rotate with the Earth. The horizon and geographical coordinates are very important to find out many satellite parameters and equations for better understanding the problems of orbital plane, satellite distance, visibility of the satellite, coverage areas, etc.

The Horizon system is 'local to the observer' or sub satellite point, because it doesn't involve any geographical coordinates. It is defined by the coordinates Azimuth and Altitude as this relation is illustrated in **Figure 3.**

1. Azimuth (Az) is measured eastward in degrees, beginning at the direction North. North is at 0 degrees, East at 90^0, South at 180^0 and West at 270^0.

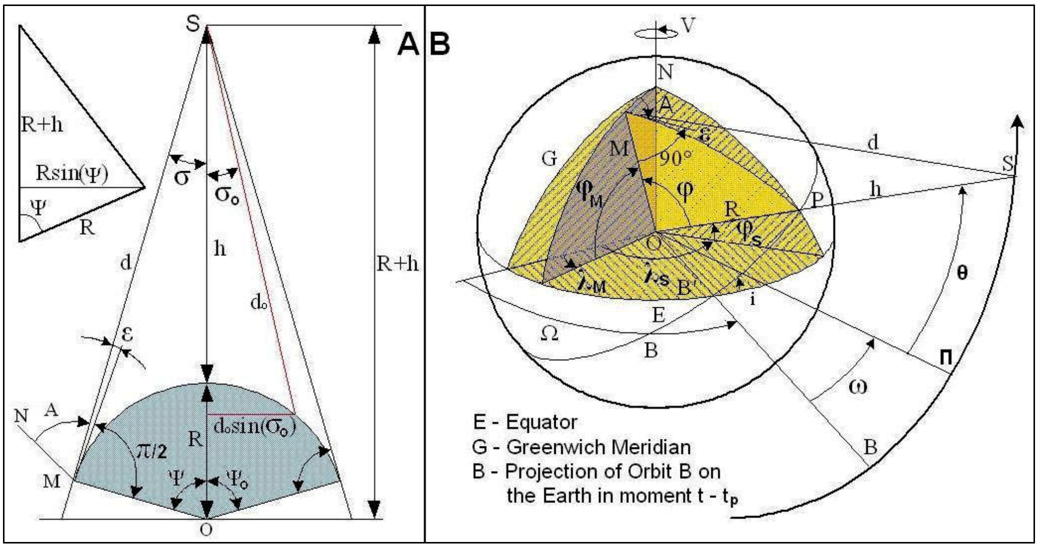

Figure 4. Geometric Projection of Satellite Orbits

2. Altitude (h or A) is measured from 0^0 to $\pm 90^0$, + is toward the zenith, the position directly above the observer head, and – is toward nadir, the position opposite the Zenith.
The coverage areas of a satellite are illustrated in **Figure 4 (A)** with the following geometrical parameters: actual altitude (h), radius of Earth (R), angle of elevation (ε), angle of azimuth (A), distance between satellite and the Earth's surface (d) and central angle (Ψ) or sub-satellite angle, which is similar to the angle of antenna radiation (δ).
The geographical and horizon coordinates of a satellite are presented in **Figure 4 (B)** with the following, not yet mentioned, main parameters: angular speed of the Earth's rotation (ν), argument of the perigee (ω), moment of satellite pass across any point on the orbit (to), which can be perigee (Π), projection of the perigee point on the Earth's surface (Π'), spherical triangle (B'ΓP), satellite (S), the Point of the Observer or Mobile (M), latitudes of observer and satellite (φM and φS), longitudes of observer and satellite (λM and λS), inclination angle (i) of the orbital plane measured between the equatorial and orbital plane and the right ascension of an ascending node angle in the moment of to (Ωo).
Otherwise, the right ascension of an ascending node angle (Ω) is the angle in the equatorial plane measured counter clockwise from the direction of the vernal equinox to that of the ascending node, while the argument of the perigee (ω) is the angle between the direction of the ascending node and the direction of the perigee.

4.1. Satellite Distance and Coverage Area

The area coverage or angle of view for each type of satellite depends on orbital parameters, its position in relation to the Land Earth Station (LES) and geographic coordinates. This relation is very simple in the case where the sub-satellite point is in the centre of coverage, while all other samples are more complicated. Thus, the angle of a GEO satellite inside its range has the following regular reciprocal relation:

$$\delta + \varepsilon + \Psi = 90^0 \qquad (17)$$

The circular sector radius can be determined by the following relation:

$$R_s = R \sin \Psi \qquad (18)$$

When the altitude of orbit h is the distance between satellite and sub-satellite point (SP), the relation for the altitude of the circular sector can be written as:

$$hs = R (1 – \cos \Psi) \quad (19)$$

From a satellite communications point of view, there are three key parameters associated with an orbiting satellite, such as follows: (1) Coverage area or the portion of the Earth's surface that can receive the satellite's transmissions with an elevation angle larger than a prescribed minimum angle; (2) The slant range (actual LOS distance from a fixed point on the Earth to the satellite) and (3) The length of time a satellite is visible with a prescribed elevation angle.

Elevation angle is an important parameter, since communications can be significantly impaired if the satellite has to be viewed at a low elevation angle, that is, an angle too close to the horizon line. In this case, a satellite close to synchronous orbit covers about 40% of the Earth's surface. Thus, from the diagram in **Figure 4 (A)** a covered area expressed with central angle (2δ or 2Ψ) or with arc (MP≈RΨ) as a part of Earth's surface can be derived with the following relation:

$$C = \pi (Rs^2 + hs^2) = 2\pi R^2 (1 – \cos \Psi) \quad (20)$$

Since the Earth's total surface area is $4\pi R^2$, it is easy to rewrite C as a fraction of the Earth's total surface:

$$C/4\pi R^2 = 0,5 (1 – \cos \Psi) \quad (21)$$

The slant range between a point on Earth and a satellite at altitude (h) and elevation angle can be defined in this way:

$$z = [(R \sin \varepsilon)^2 + 2Rh + h^2]^{½} – R \sin \varepsilon \quad (22)$$

This determines the direct propagation length between LES, (h) and (ε) and will also find the total propagation power loss from LES to satellite. In addition, (z) establishes the propagation time (time delay) over the path, which will take an electromagnetic field as:

$$td = (3.33)\ z \quad [\mu sec] \quad (23)$$

To propagate over a path of length (z) km, it takes about 100 msec to transmit to GEO. If the location of the satellite is uncertain ± 40 km, a time delay of about ± 133 μsec is always present in the Earth-to-satellite propagation path. When the satellite is in orbit at altitude (h), it will pass over a point on Earth with an elevation angle (ε) for a time period:

$$tp = (2\Psi/360)\ (t/1 \pm (t/tE) \quad (24)$$

The quotations for right ascension of the ascending node angle (Ω) and argument of the perigee (ω) are as follows:

$$\Omega = 9,95\ (R/r)^{3.5} \cos i \quad \text{or} \quad \Omega = \Omega_o + \nu\ (t – to)$$

$$\omega = 4,97\ (R/a)^{3.5}\ [5 \cos^2 i – 1/(1 – e^2)^2] \quad (25)$$

The limit of the coverage area is defined by the elevation angle from LES above the horizon with angle of view ε=0o. In this case, the satellite is visible and its maximal central angle for GEO will be as follows:

$$\Psi = \text{arc cos}\ (R \cos \varepsilon/r) – \varepsilon \quad \text{or}$$

$$\Psi = \pi/2 – \text{arc sin}\ (R/r) = \text{arc cos}\ (R/r) – \varepsilon = \text{arc cos}\ k – \varepsilon$$

$$\Psi = \text{arc cos}\ 6,376.16/42,164.20 = \text{arc cos}\ 0.15126956 = 81o\ 17’\ 58.18”$$

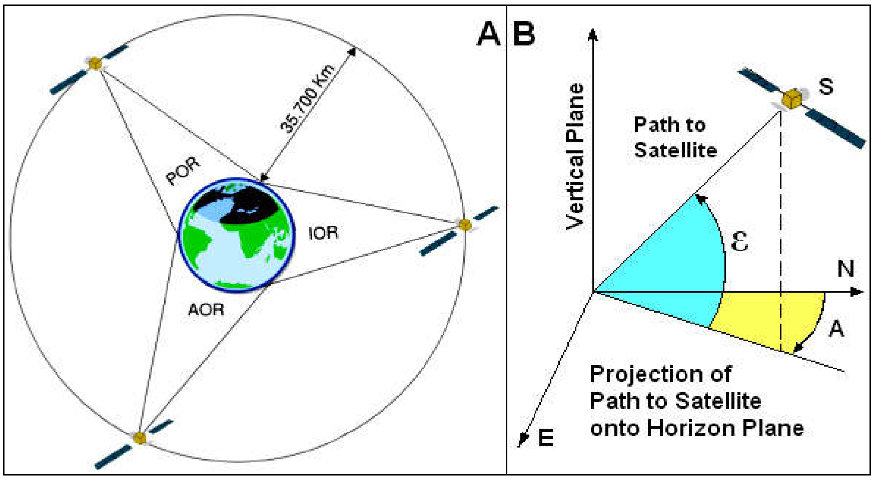

Figure 5. GEO Coverage and Look Angle Parameters

$C_{max} = 255.61 \cdot 10^6 (1 - 0.15126956) = 216.94 \cdot 10^6 \quad (km^2)$ (26)

Therefore, all MES and LES with a position above Ψ= 81o will be not covered by GEO satellites. Since the Earth's square area is 510,100,933.5 km2 and the extent of the equator is 40,076.6 km, only with three GEO mutually moved apart in the orbit by 120o it is possible to cover a great area of the Earth's surface, see **Figure 5 (A),** which shows AOR (Atlantic), IOR (Indian) and POR (Pacific) satellite coverage. The zero angles of elevation have to be avoided, even to get maximum coverage, because this increases the noise temperature of the receiving antenna. Owing to this problem, an equation for the central angle with minimum angle of view between 5o and 30o will be calculated with:

$\Psi_s = \arccos (k \cos \varepsilon) - \varepsilon$ (27)

The arch length or the maximum distant point in the area of coverage can be determined in the following way:

$l = 2\pi R (2\Psi/360 = 222.64\Psi \quad [km]$ (28)

The real altitude of satellite over sub-satellite point is as follows:

$h = r - R = 42,162 - 6,378 = 35,784 \quad [km]$ (29)

The view angle under which a GEO satellite can see Land Earth Station (LES) and Mobile Earth Station (MES) is called the "sub-satellite angle". More distant points in the coverage area for GEO satellites are limited around φ=70o of North and South geographical latitudes and around λ=70o of East and West geographical longitudes, viewed from the sub-satellite's point. Theoretically, all Earth stations around these positions are able to see satellites by a minimum angle of elevation of ε=5o. Such access is very easy to calculate, using simple trigonometry relations:

$\delta_{\varepsilon=0} = \arcsin k \approx 9^o$ (30)

At any rate, the angle (Ψ) is in correlation with angle (δ), which can determine the aperture radiation beam. For example, the aperture radiation beam of satellite antenna for global coverage has a radiation beam of 2δ=17.3o. According to Figure 2.3 (A) it will be easy to find out relations for GEO satellites as follows:

tg δ = k sin Ψ/1 – k cos Ψ = 0.15126956 sin Ψ/1 – cos Ψ/1 – 0.15126956 cos Ψ)

$\delta s = 90^0 - \Psi s = 8^0\ 42'\ 1.82''$ (31)

Differently to say, the width of the beam aperture (2δs) is providing the maximum possible coverage for synchronous circular orbit. The distance of LES and MES with regard to the satellite can be calculated using **Figure 4 (A)** and equations (13) and (22) by:

d = R sin Ψ/sin δ = r sin /cos ε (32)

The parameter (d) is quite important for transmitter power regulation of LES, which can be calculated by the following equation:

$d = \sqrt{[(R + r)^2 - 2R\ r \cos \Psi]}$ or

$d = h \sqrt{[1 + 2\ (1/k)\ (R/h)^2\ (1 - \cos \varphi \cos \Delta\lambda)]}$ or

$d = r\ [1 - (R \cos \varepsilon/r)^2]^{1/2} - R \sin \varepsilon$ (33)

Accordingly, when the position of any MES is near the equator in sub-satellite point (P) or right under the GEO satellite, then its distance is equal to the satellite altitude and takes out value for d=H of 35,786 km. Thus, every MES will have a further position from (P) when the central angle exceeds $\Psi = 81^0$, when d_{max}=41,643 km.

4.2. Satellite Look Angles (Elevation and Azimuth)

The horizon coordinates are considered to determine satellite position in correlation with an Earth observer, LES and MES terminals. These specific and important horizon coordinates are angles of satellite elevation and azimuth, which is illustrated in **Figure 4 (A and B)** and **Figure 5 (B),** respectively. The satellite elevation (ε) is the angle composed upward from the horizon to the vertical satellite direction on the vertical plane at the observer point. From point (M) shown in **Figure 4 (A)** the look angle of ε value can be calculated by the following relation:

tg ε = cos Ψ – k/sin Ψ (34)

In **Figure 6 (A)** is illustrated the Mercator chart of the 1st Generation Inmarsat space segment, using three ocean coverage areas with projection of elevation angles and with one example of a plotted position of a hypothetical ship (may also be aircraft or any mobile). Thus, it can be concluded that Mobile Earth Station (MES) at designated position ($\varepsilon = 25^0$ for IOR and $\varepsilon = 16^0$ for AOR) has the possibility to use either GEO satellites over IOR or AOR to communicate with any LES inside the coverage areas of both satellites.

The satellite azimuth (A) is the angle measured eastward from the geographical North line to the projection of the satellite path on the horizontal plane at the observer point. This angle varies between 0 and 360^0 as a function of the relative positions of the satellite and the point considered. The azimuth value of the satellite and sub-satellite point looking from the point (M) or the hypothetical position of MES can be calculated as follows:

$\text{tg } A' = \text{tg } \Delta\lambda_M - k/\sin \Psi$ (35)

Otherwise, the azimuth value, looking from sub-satellite point (P), can be calculated with the following relation:

tg A = sin Δλ/tg φ or sin A = cos φ sin Δλ cosec Ψ (36)

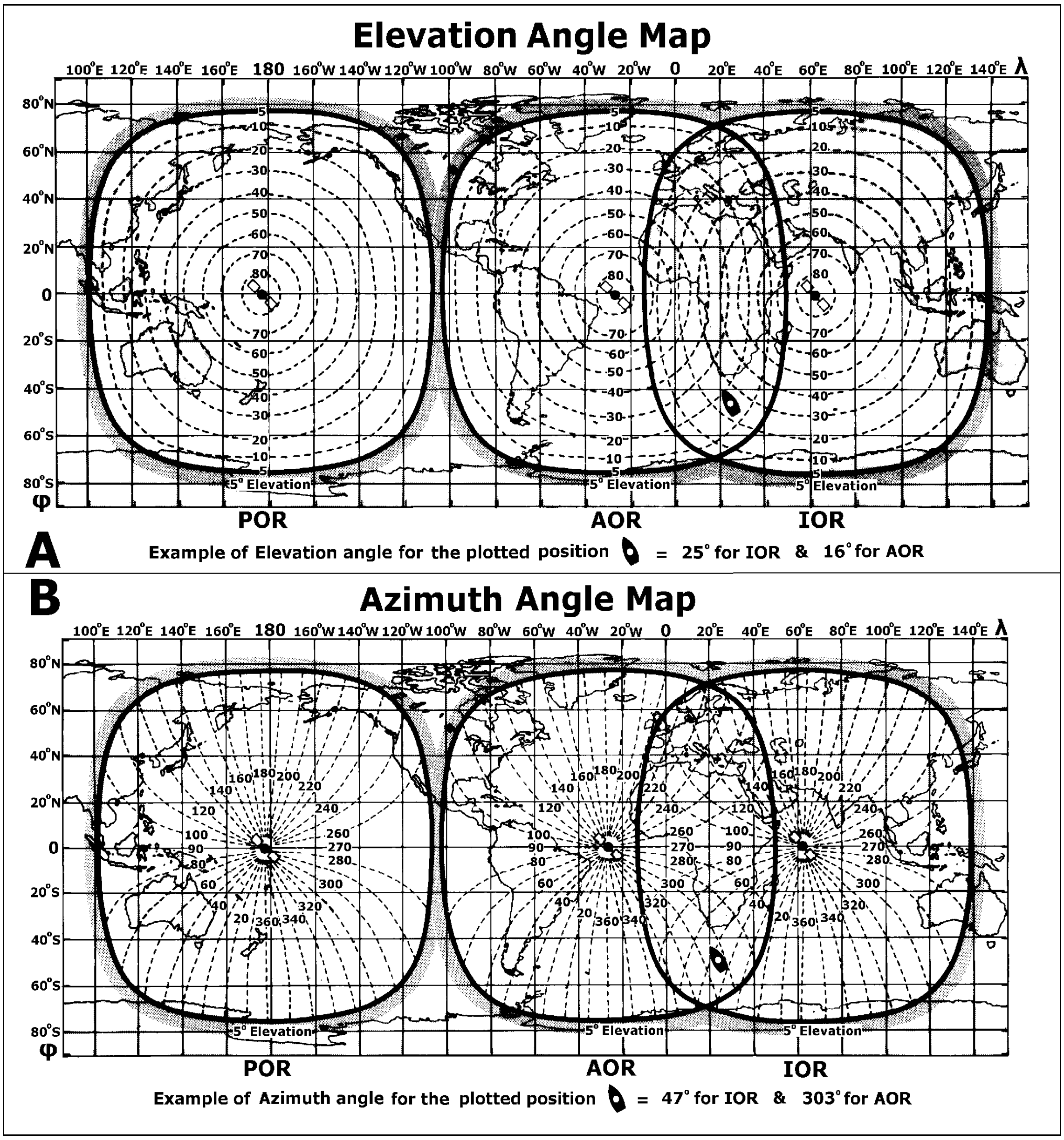

Figure 6. Elevation and Azimuth Angle Maps

In **Figure 6 (B)** is illustrated the Mercator chart of 1st Generation Inmarsat 3-satellite or ocean coverage areas with projection of azimuth angles, with one example for the plotted position of a hypothetical ship ($\varepsilon=47^0$ for IOR and $\varepsilon=303^0$ for AOR). Any mobile inside of both satellites' coverage can establish a radio link to the subscribers on shore via any LES. However, parameter (A') is the angle between the meridian plane of point (M) and the plane of a big circle crossing this point and sub-satellite point (P), while the parameter (A) is the angle between a big circle and the meridian plane of point (P). Thus, the elevation and azimuth are respectively vertical or horizontal look angles, or angles of view, in which range the satellite can be seen.

In **Figure 7 (A)** is presented a correlation of the look angle for three basic parameters (δ, Ψ, d) in relation to the altitude of the satellite. Inasmuch as the altitude of the satellite is increasing as the values of central angle (Ψ), distance between satellite and the Earth's surface (d) and duration of communication (t_c) or time length of signals are increasing, while the value of sub-satellite angle (δ) is indirectly proportional. An important increase of look angle and duration of communication can be realized by increasing the altitude to 30 or 35,000 km, while an increase in look angle is unimportant for altitudes of more than 50,000 km.

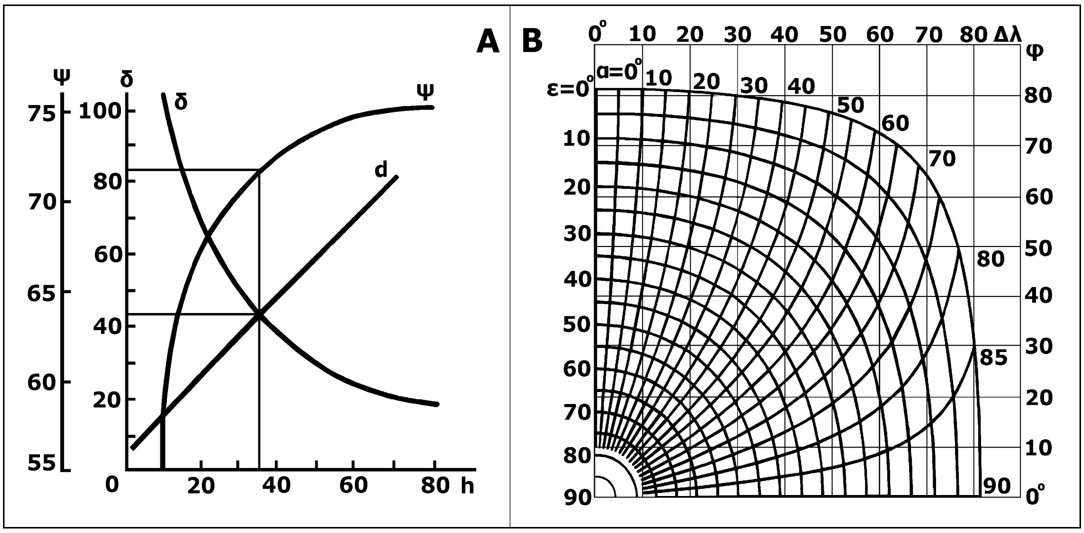

Figure 7. Look Angle Parameters and Graphic of Geometric Coordinates for GEO Network

The duration of communication is affected by the direction's displacement from the centre of look angle, which will have maximum value in the case when the direction is passing across the zenith of the LES. The single angle of the satellite in circular orbit depends on the t/2 value, which in area of satellite look angle, can be found in the duration of the time and is determined as:

$$t_c = \Psi \, t / \pi \qquad (37)$$

Practical determination of the geometric parameters of a satellite is possible by using many kinds of plans, graphs and tables. It is possible to use tables for positions of MES (φ, λ), by the aid of which longitudinal differences can be determined between MES and satellite for four feasible ship's positions: N/W, S/W, N/E and S/E in relation to GEO.

One of the most important practical pieces of information about a communications satellite is whether it can be seen from a particular location on the Earth's surface. In **Figure 7 (B)** a graphic design is shown which can approximately determine limited zones of satellite visibility from the Earth (MES) by using elevation and azimuth angles under the condition that $\delta = 0$. This graphic contains two groups of crossing curves, which are used to compare (φ) and (Δλ,) coordinates of mobile positions. Thus, the first group of parallel concentric curves shows the geometric positions where elevation has the constant value (ε=0), while the second group of fan-shaped curves starting from the centre shows the geometric positions where the difference in azimuth has the constant value (a = 0). This diagram can be used in accordance with **Figure 4 (B)** in the following order:

1) First, it is necessary to note the longitude values of satellite (λ_S) and mobile (λ_M) and the latitude of the mobile (φ_M), then calculate the difference in longitude (Δλ) and plot the point into the graphic with both coordinates (φ_M & Δλ).

2) The value of elevation angle (ε) can then be determined by a plotted point from the group of parallel concentric curves.

3) The difference value of azimuth (a) can be determined by a plotted point from the group of fan-shaped curves starting from the centre.

4) Finally, depending on the mobile position, the value of azimuth (A) can be determined on the basis of the relations presented in **Table 2.**

Table 2. The Form for Calculation of Azimuth Values

The GEO direction in relation to MES	Calculating of Azimuth Angles
Course of MES towards S & W	A = a
Course of MES towards N & W	$A = 180^0 - a$
Course of MES towards N & E	$A = 180^0 + a$
Course of MES towards S & E	$A = 360^0 - a$

Inasmuch as the position of Ship Earth Station (SES) or any MES is of significant or greater height above sea level (if the bridge or ship's antenna is in a very high position) or according to the flight altitude of Aircraft Earth Station (AES), then the elevation angle will be compensated by the following parameter:

$$x = \arccos (1 – H/R) \quad (38)$$

Where H = height above sea level of observer or MES. Let us say, if the position of LES is a height of H = 1,000 m above sea level, the value of $x \approx 1^o$. This example can be used for the determination of AES compensation parameters, depending on actual aircraft altitude. In such a way, the estimated value of elevation angle has to be subtracted for the value of the compensation parameter (x).

4.3. Satellite Track and Geometry (Longitude and Latitude)

The satellite track on the Earth's surface and the presentation of a satellite's position in correlation to the MES results from a spherical coordinate system, whose centre is the middle of Earth, is illustrated in **Figure 4 (B)**. In this way, the satellite position in any time can be decided by the geographic coordinates, sub-satellite point and range of radius. Thus, the sub-satellite point is a determined position on the Earth's surface; above it is the satellite at its zenith.

The longitude and latitude are geographic coordinates of the sub-satellite point, which can be calculated from the spherical triangle (B'ΓP), using the following relations:

$$\sin \varphi = \sin (\Theta + \omega) \sin i$$

$$tg (\lambda_S – \Omega) = tg (\Theta + \omega) \cos i \quad (39)$$

With the presented equation in previous relation it is possible to calculate the satellite path or trajectory of sub-satellite points on the Earth's surface. The GEO track breaks out at the point of coordinates $\varphi = 0$ and $\lambda = const.$

Furthermore, considering geographic latitude (φ_M) and longitude (λ_M) of the point (M) on the Earth's surface presented in **Figure 4 (B)**, what can be the position of the MES, taking into consideration the arc (MP) of the angle illustrated in **Figure 4 (A)**, the central angle can be calculated by the following relations:

$$\cos \Psi = \cos \varphi_S \cos \Delta\lambda \cos \varphi_M + \sin \varphi_S \sin \varphi_M \text{ or} \quad (40)$$

$$\cos \Psi = \cos \text{arc } MP = \cos \varphi_M \cos \Delta\lambda$$

The transition calculation from geographic to spherical coordinates and vice versa can be computed with the following equations:

$$\cos \Psi = \cos \varphi \cos \Delta\lambda \quad \text{and} \quad tg\, A = \sin \Delta\lambda / tg\, \varphi, \quad \text{respectively}$$

$$\sin \varphi = \sin \Psi \cos A \quad \text{and} \quad tg\, \Delta\lambda = tg\, \Psi \sin A \quad (41)$$

These relations are useful for any point or area of coverage on the Earth's surface, then for a centre of the area if it exists, as well as for spot-beam and global area coverage for MSC systems. The optimum number of GEO satellites for global coverage can be determined by:

$$n = 180^0/\Psi \quad (42)$$

For instance, if $\delta = 0$ and $\Psi = 81^{o}$, it will be necessary to put into orbit only 3 GEO, and to get a global coverage from 75º N to 75º S geographic latitude. Hence, in a similar way the number of satellites can be calculated for other types of satellite orbits.
The trajectory of radio waves on a link between an MES and satellite at distance (d) and the velocity of light ($c = 3 \times 10^8$ m/s) require a propagation time equal to:

$$T = d/c \quad (s) \tag{43}$$

The phenomenon of apparent change in frequency of signal waves at the receiver when the signal source moves with respect to the receivers (Earth) was explained and quantified by Johann Doppler (1803–53). The frequency of the satellite transmission received on the ground increases as the satellite is approaching the ground observer and reduces as the satellite is moving away. This change in frequency is called Doppler Effect or shift, which occurs on both the uplink and the downlink.
This effect is quite pronounced for LEO and compensating for it requires frequency tracking in a narrowband receiver, while its effect are negligible for GEO satellites. The Doppler shift at a transmitting frequency (f) and radial velocity (v_r) between the ground observer and the transmitter can be calculated by the following relation:

$$\Delta f_D = f\, v_r / c \quad \text{where} \quad v_r = dR/dt \tag{44}$$

For an elliptical orbit, assuming that R = r, the radial velocity is given by:

$$v_r = dr/dt = (dr/\Theta)\,(d\Theta/dt) \tag{45}$$

The sign of the Doppler shift is positive when the satellite is approaching the observer and vice versa. Doppler effect can also be used to estimate the position of an observer provided that the orbital parameters of the satellite are precisely known. This is very important for development of Doppler satellite tracking and determination systems.

5. Conclusion

The background theory of orbital mechanics and geospatial coordinates in satellite circular and elliptical orbits was given in this paper related to the MSC and navigation systems and for pointing of satellite tracking antenna onboard mobiles. Motion through space can be visualized using the laws described by Johannes Kepler and understood using the laws described by Sir Isaac Newton. Thus, knowledge of orbital motion and coordinates is essential for a full understanding of space operations. The orbital platform is an artificial object located in orbit around the Earth at a minimum altitude of about 20 km in the stratosphere and a maximum distance of about 36,000 km in space. The artificial platforms can have a different shape and designation but usually they have the form of aircraft, airship or spacecraft, which can provide service for many mobile applications.
Therefore, current satellite constellations are well suited for all mobile applications because of their capability to enhance coverage and support long-range mobility using mobile satellite devices and satellite antenna. All satellite transceivers onboard mobiles needs suitable satellite tracking antennas, which tracking mechanism has to point antenna to the certain satellite. At this sense, the orbital coordinates, such as elevation and azimuth look angles including geographical values of longitude and latitude are essential for precise pointing of satellite mobile antenna in focus of adequate satellite.
Satellites provide the best and very attractive alternative for commercial, military and distress communications and navigation, including mobile DVB-RCS solutions and access to the Internet. In fact, satellites are attractive in sparsely populated areas, where high bandwidth of radio cellular systems cannot be economically deployed or in impervious regions where deployment of any terrestrial facilities is not practical. Today several MSC operators provide global, regional and local coverage for all applications via both GEO and Non-GEO or Low Earth Orbit (LEO) satellites.

References

[1] Ilcev D. S. "Global Mobile Satellite Communications for Maritime, Land and Aeronautical Applications", Volume 1 (Theory) & 2 (Applications), Springer, Boston, US, 2016/2017.
[2] Evans B.G., "Satellite Communication Systems", IEE, Peter Peregrinus, London, UK, 1991.
[3] Zhilin V. A., "Mezhdunarodnaya sputnikova sistema morskoy svyazi – Inmarsat", Sudostroenie, Leningrad, Russia, 1988.
[4] Maral G. at al, "Satellite Communications Systems", Wiley, Chichester, UK, 2009.
[5] Novik L. I. at al, "Sputnikovaya svyaz na more", Sudostroenie, Leningrad, Russia, 1987.
[6] Fujimoto K., "Mobile Antenna Systems Handbook", Artech House, London, UK, 2008.
[7] Gallagher B., "Never Beyond Reach", Inmarsat, London, UK, 1989.
[8] Pratt T. at al, "Satellite Communications", John Wiley, Hoboken, US, 2002.
[9] Kadish J. E. at al, "Satellite Communications Fundamentals", Artech House, Boston, US, 2000.
[10] Kantor L.Y. " Sputnikovaya svyaz i problema geostacionarnoy orbiti", Radio i svyaz, Moskva, Russia, 1988
[11] EB, "Saturn 3 Standard-A Installation manual", EB, Nesbru, Norway, 1986.
[12] Martin H. D. at al, "Communication Satellite", AIAA, Reston, US, 2007.
[13] Noll E. M., "Landmobile and Marine Radio Technical Handbook", Howard W. Sams, Indianapolis, US, 1985.
[14] Ohmori S. at al, "Mobile Satellite Communications", Artech House, Boston–London, US/UK, 1998.
[15] Richharia M., "Mobile Satellite Communications - Principles and Trends", Addison-Wesley, Harlow, UK, 2001.
[16] Roddy D., "Satellite Communications", McGraw Hill, New York, US, 2006.
[17] Pelton J. N., "History of Satellite Communications", Springer, Boston, US, 2013.
[18] Ilcev D. S., "Global Aeronautical Communications, Navigation and Surveillance (CNS)", Volume 1 and 2, Theory and Applications, AIAA, Reston, US, 2013.
[19] Stacey D., "Aeronautical Radio Communication Systems and Networks", John Wiley, Chichester, UK, 2008.
[20] Maini A.K. at al, "Satellite Technology – Principles and Applications", Wiley, Chichester, UK. 2007.

BIOGRAPHY OF AUTHOR

Prof. Dimov Stojce Ilcev is a senior research professor at University of Johannesburg (UJ). He has three BSc degrees in Radio Engineering, Nautical Science and Maritime Electronics and Communications. He got MSc and PhD in Mobile Satellite Communications and Navigation as well. Prof. Ilcev also holds the certificates for Radio operator 1st class (Morse), for GMDSS 1st class Radio Electronic Operator and Maintainer and for Master Mariner without Limitations. He is the author of 9 academic books and more than 330 research papers in mobile Radio and Satellite CNS, DVB-RCS, Satellite Asset Tracking (SAT), Stratospheric Platform Systems (SPS) for maritime, land (road and railways), and aeronautical applications.